Kraken equation development

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How did using default insurance change the hoary banking multiplier?

- To answer this we need to compare normal banking with this insured loan method. To do that, I will use more than one type of diagram, since it expands in more than three dimensions.

So, here we start with a diagram of ordinary banking and 5% reserve requirement.

- With each iteration, 95% of the capital deposited can be loaned.
- In the next slide we will see what loans made on the basis of insuring an existing loan accomplishes.
As before, we start with an original deposit. And, as shown in dashed lines, 95% of each loan can then be loaned out.

But now, we create default insurance note (DIN) capital in addition to the loan.

- And each of those DIN backed loans in turn gets deposited.
  - And each of those deposits becomes the basis for new DIN capital.
Let’s start on a toy DIN example.

• Let us assume that we go one level deep. This means that for each conventional loan, we insure that conventional loan, and make one DIN loan from it.

• The value of that new DIN loan for each conventional loan will be assumed to be equal to the value of the insured loan. (e.g. a zero cost of insurance)

\[
m = \sum_{i_1=1}^{n} ((1 - R)^{i_1} + (1 - R)^{i_1})
\]
Decomposing the first toy equation

\[ m = \sum_{i_1=1}^{n} ((1 - R)^{i_1} + (1 - R)^{i_1}) \]

Classical banking multiplier term

The new single DIN originated loan if it were equal to the Insured loan amount.

This equation will not be reduced to simplest form because in the next steps the second term is modified and becomes quite important.
Working out the first toy equation where cost of insurance is set to zero

\[ m = \sum_{i=1}^{n} ((1 - R)^{i_1} + (1 - R)^{i_1}) \]

\[ m = \sum_{i=1}^{n} (1 - R)^{i} \]

Value of \( m \) as reserve and iterations vary

\( R = \) reserve fraction
\( n = \) iteration limit

Value of \( m \) as reserve and iterations vary

\( R = \) reserve fraction
\( n = \) iteration limit
Now let us go more than one level deep.  
We will call this “Toy 2” example.

• To go two levels deep means that for each conventional loan, we insure it, and then make a DIN loan. Then, we make one conventional loan and a new DIN loan from the DIN loan deposit.

• Again, the cost of insurance on each new DIN loan will be set to zero.

• Notice that to do this, one must nest the equations because this new layer is only created from the second term.

\[ m = \sum_{i_1=1}^{n} ((1 - R)^{i_1} + ((1 - R)^{i_1} \cdot \sum_{i_2=1}^{n} ((1 - R)^{i_2} + ((1 - R)^{i_2}))) \]
Decomposing the “Toy 2” equation

\[ m = \sum_{i_1=1}^{n} ((1 - R)^{i_1} + ((1 - R)^{i_1} \cdot \sum_{i_2=1}^{n} ((1 - R)^{i_2} + ((1 - R)^{i_2})) \right) \]

Classical banking multiplier terms

The new DIN originated loan if it were equal to the Insured loan amount.

Note that to implement it is necessary to create a new chain of conventional loans plus DIN loans for each of the DIN originated loans in the previous layer.

Refer to the previous diagrams for clarification.
Working out the “Toy 2” equation where cost of insurance is set to zero

\[
m = \sum_{i=1}^{n} ((1-R)^{i_1} + (1-R)^{i_2}) \cdot \sum_{i_2=1}^{n} ((1-R)^{i_2} + (1-R)^{i_2})
\]

\[
m = \sum_{i=1}^{n} (1 - R)^i
\]

Note scale difference

Value of \( m \) as reserve and iterations vary
\( R = \) reserve fraction
\( n = \) iteration limit
Now let us continue with “Toy 3” example.

• Again, the cost of insurance on each new DIN loan will be set to zero.

• Again, we will nest the equations, because for each loan in the second layer, we will now allocate a new standard loan in the third layer, and a new DIN based loan also in the third layer.

\[
m = \sum_{i_1=1}^{n} ((1 - R)^{i_1} + ((1 - R)^{i_1} \cdot \sum_{i_2=1}^{n} ((1 - R)^{i_2} + ((1 - R)^{i_2} \cdot \sum_{i_3=1}^{n} ((1 - R)^{i_3} + ((1 - R)^{i_3}))))))
\]
Decomposing the “Toy 3” equation

\[ m = \sum_{i_1=1}^{n} ((1 - R)^{i_1} + ((1 - R)^{i_1} \cdot \sum_{i_2=1}^{n} ((1 - R)^{i_2} + ((1 - R)^{i_2} \cdot \sum_{i_3=1}^{n} ((1 - R)^{i_3} + ((1 - R)^{i_3})))))) \]

Classical banking multiplier terms

The new DIN originated loan if it were equal to the Insured loan amount.

Again, it is necessary to create a new chain of conventional loans plus DIN loans for each of the DIN originated loans in the previous layer. Refer to the previous diagrams for clarification.
Working out the “Toy 3” equation where cost of insurance is set to zero

\[ m = \sum_{i=1}^{n}((1-R)^i + (1-R)^i \cdot \sum_{i=1}^{n}((1-R)^i + (1-R)^i) \cdot \sum_{i=1}^{n}((1-R)^i + (1-R)^i)) \]

Value of \( m \) as reserve and iterations vary

- \( R = \) reserve fraction
- \( n = \) iteration limit

Note scale difference
And so on… then some more terms

• The depth of nesting can be arbitrarily large. For this paper maximum nesting was set at 10 and all values shown are for selected values in that range.

• But of course, these equations need some adjustments to make them fit the real world better.

• The new terms to be introduced are:
  
  - \( I \equiv \) Cost of the insurance.
  - \( O \equiv \) Value of the new loan plus origination fees
  - \( T \equiv \) Tranche fraction
\( I \equiv \text{Cost of the insurance.} \)

- Cost of insuring the loan by acquiring a default insurance note (DIN) is first shown in equation 3.
- When CDS contracts (a type of DIN) were bought, they were charged based on the value of the contract.
- Consequently, the \( I \) term represents the fractional cost relative to the loan being insured.
- So, assuming that the DIN contract is equal to the face value of the loan that was issued, the \( I \) term is subtracted from 1.
- In this scheme the 1 is a placeholder for the value of the loan that was issued.
- Thus a new term is introduced to our first toy equation:

\[
m = \sum_{i=1}^{n} ((1-R)^i + ((1-R)^i \cdot (1-I))) \quad \text{“Toy 1” + } I\]
\( O \equiv \text{Value of the new loan plus origination fees} \)

- In the real world loans have origination fees. Since these are part of the transaction, they can potentially compensate for some of the cost of the insurance. We will refer to points as \( P \).
- So, the \( O \) term represents the value of the loan plus the origination fee points. Calculating this, \( O = 1 + P \).
- The \( O \) term appears in equation 4 with a discussion.
- Thus a the new \( I \) term introduced into our first toy equation is modified to include the \( O \):

\[
m = \sum_{i=1}^{n} ((1 - R)^{i_1} + ((1 - R)^{i_1} \cdot (O - I)))
\]

“Toy 1” + O & I
There is another significant modifier to this equation that comes from how bundled loans were packaged.

They were packaged in payoff fractions, or “tranches”. Typically, a bundle of loans would be divided into three sections. The first tranche would be paid first. Until everyone in the first tranche was paid, nobody was paid in the second. The second took precedence over the third.

Typically, only the first tranche was insured. So, the fraction of loans representing that tranche is the limit what can be acquired as insurance.

The $T$ term appears in equation 5 with a discussion.

Thus with the addition of a new $T$ term into the first toy equation results in:

$$m = \sum_{i_1=1}^{n} \left( (1-R)^{i_1} + ((1-R)^{i_1} \cdot (O-I) \cdot T) \right)$$

“Toy 1” + O, I & $T$