Banking multiplier background
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December, 2011

\[ m = \sum_{i=1}^{n} (1 - R)^i \]
We begin with the classical banking multiplier

• The classic banking multiplier starts with the concept of reserves.

• Reserves allow new money to be created by banks through the issuance of loans. This happens because the requirement for physical money representation is eliminated.

• The banking multiplier is taught as:

\[ m = \frac{1}{R} \]  \hspace{1cm} (1)

where \( R \) = capital reserve fraction
How does banking create money?

- To understand this we will use a simplified system with Zeke and Jane and two banks, bank 1 and bank 2.
- Zeke borrows from Bank 1 and deposits to Bank 2.
- Jane borrows from Bank 2 and deposits to Bank 1.
- These banks have a 5% reserve requirement.

- We will start with an initial deposit of $100 into Bank 1. With 5% reserve, Bank 1 can loan $95 to Zeke.
What happens with the first loan of $95?

- Zeke deposits his newly created $95 into Bank 2.

- So now Bank 2 can loan Jane 95% of that new deposit originating from Zeke’s loan he got from Bank 1. 95% of $95 = $90.25
And so the $90.25 Jane deposits into Bank 1 becomes the basis for another loan, and the cycle repeats.

- Jane deposits her newly created $90.25 into Bank 1.

- So now Bank 1 can loan Zeke another 95% of that $90.25 new deposit originating from Jane’s loan she got from Bank 2. 95% of $90.25 = $85.74
We can visualize this series.

New money

Bank 1

95% loan to Jane deposited in Bank 1

Bank 2

95% loan to Zeke deposited in Bank 2

Each time a loan is made, it becomes a new deposit, and adds to the capital base of a bank.
$100.00

| $ 95.00 | $ 56.88 | $ 34.06 | $ 20.39 | $ 12.21 | $ 7.31 | $ 4.38 |
| $ 90.25 | $ 54.04 | $ 32.35 | $ 19.37 | $ 11.60 | $ 6.94 | $ 4.16 |
| $ 85.74 | $ 51.33 | $ 30.74 | $ 18.40 | $ 11.02 | $ 6.60 | $ 3.95 |
| $ 81.45 | $ 48.77 | $ 29.20 | $ 17.48 | $ 10.47 | $ 6.27 | $ 3.75 |
| $ 77.38 | $ 46.33 | $ 27.74 | $ 16.61 | $ 9.94  | $ 5.95 | $ 3.56 |
| $ 73.51 | $ 44.01 | $ 26.35 | $ 15.78 | $ 9.45  | $ 5.66 | $ 3.39 |
| $ 69.83 | $ 41.81 | $ 25.03 | $ 14.99 | $ 8.97  | $ 5.37 | $ 3.22 |
| $ 66.34 | $ 39.72 | $ 23.78 | $ 14.24 | $ 8.53  | $ 5.10 | $ 3.06 |
| $ 63.02 | $ 37.74 | $ 22.59 | $ 13.53 | $ 8.10  | $ 4.85 | $ 2.90 |
| $ 59.87 | $ 35.85 | $ 21.46 | $ 12.85 | $ 7.69  | $ 4.61 | $ 2.76 |

**Table of deposits to banks 1 and 2**

In this table, \( n = 70 \) and \( R = 5\% \)
Mathematically, the banking multiplier \((m)\) is a summation

\[ m = \sum_{i=1}^{n} (1 - R)^i \]  

(2)

- where \(R\) = capital reserve fraction
- \(i\) = iteration number on loans/deposits
- \(n\) = iteration limit

This equation has an asymptote at equation 1.

\[ m = \frac{1}{R} \]  

(1)
How does this equation behave?

\[ \sum_{i=1}^{10} (0.95)^i = 8.623998154 \]

\[ \sum_{i=1}^{20} (0.95)^i = 13.18876747 \]

\[ \sum_{i=1}^{40} (0.95)^i = 17.55826903 \]

\[ \sum_{i=1}^{80} (0.95)^i = 19.68620789 \]

Etc.
We can render this banking multiplier ($m$) as an isosurface

- This isosurface plot shows how the money multiplier varies as iterations ($n$) go from 1 to 100.
- The reserve ($R$) parameter starts at 2% and increases to 10%.
- Most reserves in the USA and EU are around 5% to 7%.
- In the GFC, some formal reserves dropped as low as 2.4%. (Outside of central banks.)

$$m = \sum_{i=1}^{n} (1 - R)^i$$